# An Empirical Verification on the Performance of Black-Scholes option Pricing Model in Nigerian Stock Market

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**Abstract:**
This paper compared the market prices and the theoretical prices (BSOPM) of two Nigerian Stock Market Indices namely Coca-Cola company Plc and Dangote Cement listed in the Nigerian Stock Exchange. The data for this study were obtained from https://ng.www.investing.com/equities/cocacola-bottle-historical-data,and https://ng.www.investing.com/equities/dangcem-historical-data. T-Test for paired sample was used as a method of analysis to find whether there is any significant difference between the Market Price and the theoretical (BSP) price and the result revealed that there is a significant difference since the null hypothesis was rejected (P-value <0.05) and there is also a correlation between the Market Price and the Black-Scholes price. The study further revealed that prices of the two companies were moving up and down leaving the investors with either options of continuity or to stop investing with the two companies.

**Key words:** Options, Black-Scholes, Stock Prices, pairwise t-test.

## 1.0 Introduction:

In 1973, Fisher Black and Myron Scholes developed the theoretical model for the pricing of options which can reduce the price of call and put options depending on the relevant factor like the spot value of the underlying, strike price of underlying and the Risk-free rate of return available in the market.

This model of option pricing is based on the fundamental that in the future, the price of option contract either increases or decreases based on the spot price of the underlying asset. The famous Black-Scholes model is the commonly accepted model for pricing claim in financial industry. Since Black-Scholes published their articles on option pricings 1973, there had been vast explosions of theoretical and empirical investigation on option pricing. While Black-Scholes assumption of Geometric Brownian Motion (GBM) still maintained most in option pricing models, the possibility of alternative distribution hypothesis was also raised later. The main assumption of the model is the riskless interest rate assumed to be constant, and the stock price processes a geometric Brownian motion which implies stock returns are independent. The Black -Scholes formula is valid under more widely assumption of stock price process. Hence, in this paper, we applied the Black-Scholes model to estimate the option premium in order to calculate the option prices to see whether the model is effective or not in Nigeria stock market and compare, the market prices of the stocks against the Black-Scholes option price.

## 2.0 Literature Review1:

Cox, et al (1979) derived the tree methods of pricing options, based on risk-neutral valuation, the binomial option pricing European option prices under various alternatives, including the absolute diffusion pure-jump, and square root constant elasticity of variance method.

Macbeth and Merville (1979), conducted a comparative analysis of the real market prices of call option with the prices predicted by Black and Scholes 1973. The study attracted researchers from other field to use these models in the related price prediction. Years later, the Black-Scholes model has been widely used in different fields ranging from business (Corrado and Su 1996) to construction of projects (Barton and Lawryshyn 2011).

Macbeth and Merville (1980) applied the model to test the Cox call option valuation for the constant elasticity of variance diffusion process against the Black-Scholes model. This study observed the movement of common stock prices in line with the constant elasticity variance diffusion process. This result explored a new horizon for the capital market investor predict the price of common stock along with some other financial instrument.

Chesney and Scott (1989) applied the Black-Scholes model to hedging the risk against underlying securities. During the same period, the model used to quantify the more specifically, here the model was used to quantify the tradeoff between providing the generalizing training as well as the specific skill training as can be observed from Miller (1990).

Frino and Khan (1991) conducted cross-sectional experiments of the pricing technique using the historical data. His research found out the significance of this model and stated that the Black–Scholes model cannot be rejected.

Gruchy (1991) noted that Black-Scholes formula still holds even though the underlying assets returns follow an auto correlated Ornstein- Hollenbeck process.

Kim, et al (1997) analyzed the effect of implied volatility on option pricing models for at-the money put options. The study found out the inference that the implied volatility estimates derived from the BSOPM European model were almost similar to those derived from the other more complex pricing methods.

Genkay and Salih (2003) observed that the BSOPM model pricing errors are bigger in the deeper out-of-the-money options, and volatility increases the mispricing. This result stated that the BSOPM model is not the appropriate pricing tool in high volatility.

Bonz and Angeli (2010) tested the applicability and relevance of the Black–Scholes model for price stock index options. They determined the theoretical prices of options under the BSOPM model assumptions and then compared these prices with the real market values to find out the degree of variation in two different time zones. They finally concluded that BS model performed differently in the period before and after the financial crisis.

Mishra (2012) examined in his paper the exactness of choice in estimating models to value Nifty Indexed Futures trading on National Stock Exchange (NSE) of India. The paper endeavored to address the issues identified with undervaluing of Nifty options by virtue of negative cost of convenience in future market. In this examination, the choices are cited utilizing both Black–Scholes equation and Black–Scholes formula and the results concluded that the Black's formula deliver preferred option over utilization of Black and Scholes formula. From the examination of blunders, it is confirmed that Black model delivers less mistake than that of Black–Scholes display and therefore utilization of Black model is more fitting than that of Black-Scholes model for valuing Nifty options.

Nilakantan and Jain, (2014) Found in their study in the context of Indian Stock market that the Black-Scholes model suffers from various deficiencies. They concluded that modified Black-Scholes Model is not able to produce efficient results for NIFTY index option in case of At-the-money, Out of the Money and Deep Out-of-the-Money options. In most of cases call options are under prices by the Black-Scholes model. The Black-Scholes Model under prices high volatility stock. It pays low reward for the stock that has high volatility.

Sharma and Arora, (2015) tested the relevance of Black-Scholes Model in the Indian Stock market for the Option prices by using the model to calculate the theoretical Option Prices using the equation and then comparing it with the actual values. All the necessary assumptions have been taken into consideration in this research as required by the model for option price calculation. The research concluded that the Black Scholes model values were not relevant to the market values of the stock options. The findings also showed that there is a need to explore other impacts on the pricing of the stock options than the Black Scholes Model.

Del Giudice et al. (2016) conducted a review of Black-Scholes. An overview of this of the qualitative approach based study reveals that most of the application of the Black-Scholes model lies in the Business study sector focusing on the financial markets including investment in research and development especially in pharmaceutical company (McGrath 1997; 2004), customer relationship management (Makle, et al 2014) assessment of bonds and derivative (Singh 2014) management and evaluation of intangible assets (Park, et al 2012).

Kumar and Agrawal, (2017) stated that the Black-Scholes Model suffers from a pricing error at the deeper out-of-the money options, which is greater, compared at the near out-of-the money options and this error increases as the volatility increases. The Black-Scholes model suffers from an error of mispricing options considerably and this error of mispricing increases as the moneyness and volatility increases. The Black-Scholes model overprices short-term options and underprices long-term options. The Black-Scholes model exhibits pricing errors on several parameters. The Black-Scholes model under prices in-the money options and overprices out-of-the-money options. The pricing errors are comparatively lesser in the modified BS model compared to the present one. The Black-Scholes model suffers from various deficiencies. It was understood that the modified BS Model would not be able to produce efficient results for NIFTY index option in the case of At-the-money, Out-of the Money and Deep Out-of-the-Money options. The Black-Scholes Model under prices stocks with high volatility and pays low reward for these stocks.

McKenzie and Subedar (2017) concluded in their report that BSOPM is relatively accurate. They concluded that the Black–Scholes model is significant at 1 per cent level in estimating the probability of an option.

Ugomma, et al evaluated the performance of the Black Scholes Option Pricing Model (BSOPM) to the credibility of the price stock index options, where the theoretical values of the Lehman Brothers of 2008 were calculated under the Black Scholes Model. The data collected were divided into two different windows; the normal trading days and the turbulent days built around the bankruptcy of Lehman Brothers in 2008.The empirical result showed that the Black Scholes model performed differently in the normal days and turbulent days.

This brief literature review have shown that Black-Scholes Model have a number of different application. In some cases, the results are in line with the model predictions while in other cases; it seems there are some discrepancies. Nevertheless, the model has the capability to examine various type of valuation of derivatives in different markets

## 3.0 Materials and Method:

### 3.1 The Black-Scholes Model

We are now able to derive the Black-Scholes for a call option on a non-dividend paying stock with strike price $K$ and maturity $T$. We assumed that the stock price follows a Geometric Brownian Motion, so that;

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t \qquad (1)$$

Where $W_t$ is a standard Brownian Motion. We also assumed that interest rates are constant so that 1 unit of currency invested in the cash account at time $t=0$ will be worth $B_t = \exp(rT)$ at time $t$. Let $C(S_t, T)$ be the value of the call option at time $t$.

By Ito's lemma, we know that;

$$dC(S_t,T) = \left(\mu S_t \frac{\partial C}{\partial S_t} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2}\right)dt + \sigma S_t \frac{\partial C}{\partial S_t} dW_t \qquad (2)$$

Let's now consider a self-financing trading strategy where at each time t, we hold $x_t$ and $y_t$ units of the stock. Then, $P_t$ the time t value of this strategy satisfies

$$P_t = x_t B_t + y_t S_t \qquad (3)$$

We will choose $x_t$ and $y_t$ in such a way that the strategy replicates the value of the option. The self-financing assumption implies that;

$$dP_t = x_t dB_t + y_t dS_t \qquad (4)$$

$$= x_t B_t\, dt + y_t\left(\mu S_t\, dt + \sigma S_t\, dW_t\right)$$
$$= \left(r x_t B_t + y_t \mu S_t\right)dt + y_t \sigma S_t\, dW_t \qquad (5)$$

Note that Equation (4) is consistent with our earlier definition of self-financing. In particular, any gains or losses on the portfolio are due entirely to gains or losses in the underlying securities, that is, the cash-account and stock, and not due to changes in the holding $x_t$ and $y_t$. Returning to our derivation, we equate the term in Equation (2) with the corresponding terms in Equation (5) to obtain;

$$y_t = \frac{\partial C}{\partial S_t} \qquad (6)$$

$$r x_t B_t = \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} \qquad (7)$$

If we set $C_0 = P_0$, the initial value of our self-financing strategy, then, it must be the case that $C_t = P_t$ for all t. Since $C$ and $P$ have the same changes. This is true by construction after we equated the terms in Equation (2) with the corresponding terms in Equation (5).

If we substitute Equation (6) and Equation (7) into Equation (8), we obtain;

$$rS_t \frac{\partial C}{\partial S_t} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} - rC = 0 \qquad (8)$$

In order to solve Equation (8), some necessary boundary conditions were also provided.

In the case of our call option, those conditions are;

(i) $C(S_t, T) = Max(S_t - K, 0)$

(ii) $C(0,T) = 0$ for all t

(iii) $C(S_t,T) \to S_t$ as $S_t \to \infty$

The solution to Equation (8) in the case of our call option is given by;

$$C(S_t,T) = S_t N(d_1) - Ke^{-rT} N(d_2); \quad (9)$$

$$d_2 = d_1 - \sigma\sqrt{T}; \quad (10)$$

were,

$C$ is the Call Premium,

$S_t$ is the current stock price,

$T$ is the time to maturity,

$K$ is the strike (exercise) price,

$r$ is the risk-free interest rate,

$N(\bullet)$ is the cumulative distribution function of a standardized normal distribution

$e$ is the exponential function.

The most interesting feature of Equation (8) is that μ in Equation (1) does not appear anywhere. We would note that the Black-Scholes Partial Differential Equation (PDE) would also hold if we had assumed that $\mu = r$. However, if $\mu = r$, then, investors would not demand a premium for holding stock. Since, this would generally hold if investors were risk-neutral, this method of derivatives pricing came to be known as risk-neutral pricing.

### 3.2 Method of Data Analysis:

#### 3.2.1 Computation of Black-Scholes Call Option Pricing Model

In order to obtain the theoretical value (price) of the call, we first collect all required data in Black-Scholes formula from NSE and then apply them in the BSOPM (Black-Scholes option pricing model) using Equation (9) and determine the variations between the model value and the actual market price using Equation ()

#### 3.2.2 Computation of Price Volatility

To determine the historical volatility, log of yearly returns shall be calculated using moving average method given as;

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \quad (11)$$

The yearly standard deviation is given by;

$$\bar{r}_t = \frac{\sum_{t=1}^{n}\left(\frac{S_t}{S_{t-1}}\right)}{N}, \quad (12)$$

$$S_d^2 = \frac{\sum\left(r_t^2\right) - \left(\underline{r}_t\right)^2}{n(n-1)} \quad (13)$$

and

$$Sd = \sqrt{\frac{\sum r_t^2 - (\bar{r}_t)^2}{n(n-1)}} \quad (14)$$

The yearly historical volatility is given by;

$$\sigma = Sd\sqrt{T} \quad (15)$$

Were,

T = 250 trading days

#### 2.2.3 The Pair wise t-Test for Mean Difference

This test is used to compare two population means having two samples in which observations in one sample can be paired with observations in the other sample.

In this paper, we used this test to compare the market value (stock prices) and the theoretical prices (BSOPM) where the two samples can be paired as one observation. Here, we assumed the null hypothesis of no significant difference between the means of the market prices of the stock and Black-Scholes prices of the same stock at 5% level of significance.

The pair wise t-test used in this study is given as

$$t = \frac{\bar{d}}{SE\left(\bar{d}\right)} \qquad (16)$$

where,

$$\bar{d} = \frac{\sum d_i}{n} \qquad (17)$$

$$SE\left(\bar{d}\right) = \frac{S_d}{\sqrt{n}} \qquad (18)$$

and

$$S_d = \sqrt{\frac{n\sum d_i^2 - \left(\sum d_i\right)^2}{n\left(n-1\right)}} \qquad (19)$$

We note Equation (3.8) follows a t-distribution with $n-1$degrees of freedom.

## 4.0 Empirical Evidence:

### 4.1 Graphical Presentation of Original Stock Prices from 2013 to 202

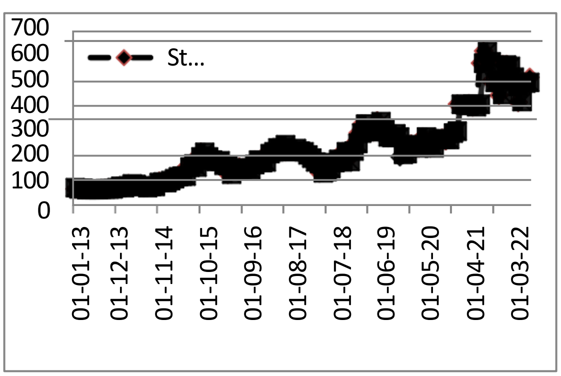


**Fig 4.1a** The Plot of Coca-Cola PLC Stock price for 2013 to 2022

From Fig 4.1a, we observed steady fall decline in price of Coca-Cola company between January 2013 to September 2015 but

increased slightly January 2016 and

maintained somehow price increase till May 2018.The company's price appreciated between September 2018 and maintained steady price increase till January 2022 before price fluctuations between May 2022 to December 2022. From the discussion, we observed that there are fewer investors between 2013 to 2018 but many investors between mid-2018 to 2022, consequently, as stock prices moves up and down, their

volatility can have positive or negative impact on consumers and businesses

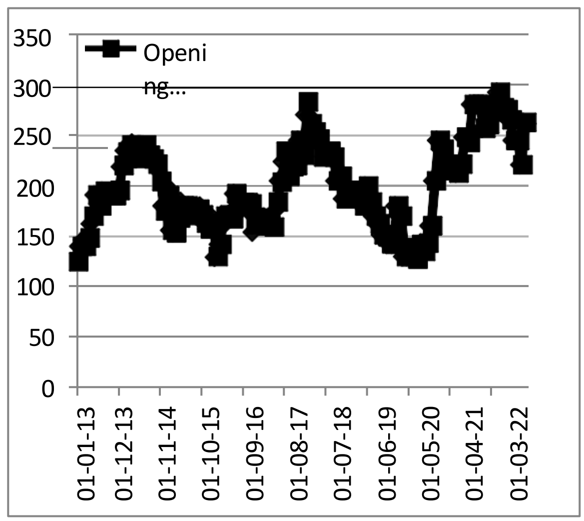


**Fig 4.1b** The Plot of Dangote Cement Stock price for 2013 to 2022

Fig 4.1b shows series of price changes (up and down movements) between January 2013 to May 2017. Dangote Cement observed price increase from June 2017 to May 2018, price decline September 2018 and fluctuates until 2022. This means that there are fewer investors when the prices fall and more investors when the prices rise. Hence, there is no steady volatility on the impact of the consumers and the investors.

***Table 4.1 Descriptive Statistics of Stock Prices of Coca-Cola Company and Dangote Cement PLC***

| Stock | Sample Size | Mean | Variance | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Coca-Cola | 119 | 0.9893 | 0.0114 | 0.2992 | 1.4456 |
| Dangote Cement | 119 | 0.9943 | 0.0136 | 0.1147 | 1.0609 |

Table 4.1 shows the descriptive statistics of two companies enlisted in the Nigerian Stock Exchange namely the Coca-Coca Bottling Company PLC and Dangote Cement. The result show that the mean (price returns) for Coca-Cola is 0.9893 (98.9% price returns) and 0.9943 (99.4% price returns) for Dangote Cement. The output further showed that their variances are 0.0114 and 0.0136 for Coca-Cola and Dangote Cement respectively. Consequently, Coca-Cola Company is associated with low and a lower return in investment than that of Dangote Cement. The output also revealed that the skewness of both companies is positive showing that the companies generate frequent small losses and few extreme gains under the period of study. Hence, the stocks tend to be good for conservative investors who have less risk tolerance.

### *4.2 Determination of significant difference between the Actual Stock prices and Black-Scholes Call Prices*

Table 4.2 The output of Actual Stock Price of Coca-Cola Cement and Black-Scholes Call Price

| Pairwise | Sample Size | Mean | Standard deviation | Standard Error | df | t-critical | correlation | Significance (2 tailed) |
|---|---|---|---|---|---|---|---|---|
| **Coca-Cola-Black-Scholes** | 119 | 88.1062 | 53.8817 | 4.9393 | 118 | 17.838 | 1.0000 | 0.000 |
| **Dangote-Black-Scholes** | 119 | 198.3883 | 42.0143 | 3.8515 | 118 | 51.510 | -0.119 | 0.000 |

The output showed that both P-values are less than 0.05. Hence, we reject the null hypothesis and conclude that there is statistically difference between the market price of Coca-Cola, and the Black-Scholes Call price and Dangote Cement and Black-Scholes Call Price. The output also showed positive perfect correlation (1.00) between the market price and the Black-Scholes Call price and negative correlation between the market price of Dangote Cement and the Black-Scholes call prices. This means that there is mispricing between the stock prices and the theoretical price of Dangote Cement between 2013 and 2022.

### 4.3 Determination of Stock Price Volatility between Coca-Cola Company and Dangote Cement

| Stock | Sample Size | Standard Deviation | Volatility | % Volatility | Remark |
|---|---|---|---|---|---|
| **Coca-Cola** | 119 | 0.9946 | 1.685495 | 16.85 | Low Risk |
| **Dangote Cement** | 119 | 0.9971 | 1.845189 | 18.45 | Low Risk |

Since volatility is the rate, at which the price of stock increases or decreases over a particular period. The higher the stock price volatility, the higher the risk in investment. From Table 4.3, we observed that the stocks volatilities of Coca-Cola Company and Dangote Cement are approximately 17% and 18% respectively. This implies that the risk of investors investing in these two companies is low and almost the same over the years of investment. Therefore, the investors are advised to continue buying shares with these companies they have low risk in average returns in their investments over the years under study.

## 5.0 Conclusion:

From this study, we observed that the stock prices of Coca-Cola company and Dangote Cement fluctuates over the years and there is statistically difference the market price and the Black-Scholes prices over the years of study and that correlation existed between the market price and the theoretical price of Coca-Cola and Dangote Cement for the perod of study.

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