

Mathematical Modeling of Blood Flow through Arteries in the Presence of Magnetic Field with Porosity

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Abstract:

The present paper explores a comparative study of two non-Newtonian mathematical models with porosity. The first mathematical model is based on the Power law model with porosity, and the second mathematical model is developed on the Maxwell principle having porosity. An attempt has been made to study how the shear-thinning viscoelastic affects the rheology of blood under the magnetic field and porosity, when porosity through the vessel wall is taken into account with magnetic field in the blood arteries. The solution has been obtained by solving the differential equations occurring during the mathematical modeling by Finite Difference Method.

Keywords: shear thinning viscosity, Rheology, Porosity

1. Introduction

In human physiology, we come across the vessels where tissues are present, and tissues function for secreting nutrients. This creates an effect of porosity on blood flow. In the present paper, we are studying the effects of a magnetic field with porosity.

A large number of researchers have already worked on a similar topic. Here we will list some important researchers' work which is closely related to our work.

Cribber (1965) has developed the model taking into account pressure gradient. Rabbis and Kaferen (1982) have studied a benchmark work related to the topic by developing a mathematical

model taking into account the pressure gradients. Lipsch and Moravee (1984) developed a mathematical model for pulsatile blood flow in various branches of arteries in the presence of a magnetic field. Chakra arty (1986) developed two different mathematical models which were majorly based on limitations of oscillatory conditions of blood flows. Rind et al. (1987) had studied the computational study of pulsatile flow of blood taking into account blood as a composition of RBC, WBC, and plasma. Rodkiewicz et al. (1990) compared various mathematical models of blood. They have studied various mathematical models by solving various numerical techniques. Duttal et al. (1992) studied taking into account blood as a particle-fluid suspension. Brookshier and Tarwell (1993) developed a model for wall shear stress of blood vessels. Sharma and Kapur (1995) formulated the blood flow problem by using finite element method. Tarbel (1996) compared two different rheological models. Korenga et al. (1998) studied the bio-chemistry of blood flowing in human arteries. Rachev et al. (1998) showed how blood vessels take adaptation during transverse magnetic field. Naduvinamani et al. (2002) studied the squeezing lubrication of film by taking anisotropic rectangular plate taking porosity. Farah et al. (2002) studied the two models by taking into account of porosity and showed their results by solving different computational methods. Chein Chao (2003) studied the heated hosted cylinder. Michel et al. (2004) studied the transportation of reactive solutions in porous media. Nicholson and Petropoulos (2006) have made an investigation of branches of arterial flows. Banyal and Devnath (2006) studied the effects of flow of blood

through arteries by taking into account of periodic body acceleration. Raddaich and Prasada (2007) solved the convective diffusion problem by the finite element method and also compared their results by solving the finite volume method.

Tarchie (2008) studied the effects of porous media of different layers of flows. Nakayama and Kashwon (2008) developed a general heat transfer model. Valochi (2008) studied the effects of porous media on mixing-controlled reactions. Thomas and Houle (2009) developed a mathematical model applicable to a surface sand filter. Michel et al. (2010) developed a two-phase model for reactive solute transport in porous media. Mansourish and Shoki (2011) studied the effects of salt concentration on evaporation from porous media. Khalid and Vafai (2012) studied how patterns in human blood flow. Dabiri and Mar (2013) simulated the behaviour and background of blood flow. Dabiri et al. (2014) studied quantitative measurement of ostraca swimming. Dabiri (2015) studied vortex-enhanced propulsion. Dabiri et al. (2016) predicted the basis of ecological and invasive.

Tenophor and Witlessly (2017) used turbine machines for fish schooling as a basis of vertical axis. Gallegos et al. (2018) studied habitat alteration by species ranging from microbes to jellyfish. Nawroth (2019) developed a mathematical phenomenon for effective fluid interaction. Costello et al. (2020) studied a comparative study of swimming performance. Farewall and Dabiri (2021) studied a Lagrangian approach to identifying vortex. Pinch et al. (2022) developed a mathematical model for the behavior of seven co-occurring jellyfish.

2. Governing Equations for the Blood

Casson (1959) developed the equation of the form

$$\tau^{1/2} = n_0 \gamma^{1/2} + \tau^{1/2} \quad \dots \quad (1)$$

$$\gamma = 0, |\tau| < \tau_1 \quad \dots \quad (2)$$

Where τ is the Shear Stress γ represents the stress

and n is the casson viscosity.

$$\tau = m \gamma^n \quad \dots \quad (3)$$

$$\tau + \tau_P = \frac{n_0}{1 + (\gamma T)^v} \quad \dots \quad (4)$$

$$\tau = \sum_{p=1}^n \tau_p + n\gamma \quad \dots \quad (5)$$

3. Mathematical Modeling of the Problem

Here we present a mathematical model by treating blood as a homogeneous incompressible fluid by taking the electrically charged and Porosity inconsideration, and flow takes Place in an Anisotropic mode.

The Equations are

$$\frac{dw}{dt} + u \frac{dw}{dr} + w \frac{dw}{dz} = - \frac{1}{P} \frac{dp}{dz} - \frac{1}{Pr} \frac{d}{dr} (\gamma \tau) - \frac{6}{e} B^2 w = \frac{\mu}{ex} w \quad \dots \quad (6)$$

$$\frac{du}{dr} + \frac{dw}{dz} + \frac{\mu}{y} = 0 \quad \dots \quad (7)$$

where u and w are velocity components in the r and z directions, respectively. P is pressure and μ is density, e is the electrical conductivity and B is the magnetic field intensity. The magnetic field is such that induced magnetic field is neglected.

The boundary conditions are

$$\frac{d\omega}{dr} = 0 \quad \mu = 0 \quad \alpha \gamma = 0$$

$$W=0 \quad u = \frac{dr}{dt} \quad \alpha \gamma = \gamma = R(t, z) \quad \dots \quad (9)$$

Facing the difficulties due to the moving boundary a transformation

$z = r/(R(l, z))$ is introduced. The transformed governing equations become

$$\frac{dw}{dr} + \left(\frac{u}{r} - \frac{\epsilon_1}{R} \right) \frac{dw}{dz} - \frac{w}{r} \left(\frac{du}{ds} + \frac{u}{s} \right) = - \frac{1}{P} \frac{dP}{dz} - \frac{1}{PR} \frac{d\tau}{ds} - \frac{\tau}{PRs} - \frac{6}{PRs} B^2 w \frac{u}{\omega} \quad \text{Or} \quad \frac{dw}{dr} = - \frac{1}{P} \frac{dP}{dz} - \frac{1}{PR} \frac{d\tau}{ds} - \frac{\tau}{PRs} + \left(\frac{s}{R} \frac{dr}{dt} - \frac{\mu}{y} \right) \frac{d\omega}{ds} + \frac{d\omega}{dr} \left(\frac{d\mu}{ds} + \frac{\mu}{s} \right) - \frac{6}{P} B^2 w - \frac{\mu}{K} \omega \quad \dots \quad (10)$$

$$\mu = \frac{dR}{dr} \frac{dP}{dz} \quad Z = s \quad \int_0^s \omega ds \quad \int_0^s \epsilon \omega d\epsilon - \frac{f_0}{1} \int_0^s \epsilon \omega d\epsilon + \frac{1}{s} \frac{dr}{dt} \int_0^s \omega ds \quad \dots \quad (11)$$

Where $\frac{dR}{dP}$ represents the elastic response of the artery and experimental values for this are

available [White(1991)]. The transformed boundary conditions are

$$\frac{d\omega}{ds} = 0 \quad \mu = 0 \quad \alpha\epsilon = 0$$

.....12

$$\omega = 0 \quad \mu = \frac{dr}{dt} \quad \alpha\epsilon = 1$$

.....13

For the Solution of flow problem of a power law fluid equation (3) can be written as

$$r = -m \left(\frac{dm}{dr} \right)^{n-1} \frac{dm}{dr} \quad \text{.....(14)}$$

Where $\left(-\frac{dw}{dr} \right)$ is the shear rate for one-dimensional tube flow. The equations (9) and (14) are transformed into new Co-ordinate system such that.

$$r = -\frac{m}{r} \left| \frac{dm}{ds} \right| \frac{dm}{dz} \quad \text{..... (15)}$$

The equations (4) and (5) are transformed into the new co-ordinate system as given blow.

$$\frac{drp}{dr} = \frac{rp}{Tp} + \frac{s}{R} + \frac{dr dp}{dt ds} + \frac{Tp}{(1+\gamma rp)\gamma} \quad \text{.....(16)}$$

$$r = \sum_{p=1}^0 rp + np\gamma \quad \text{.....(17)}$$

The momentum and continuity equations (14) and (11) can be solved subject to the boundary conditions (12) and (13) to complete the flow field of non-Newtonian fluid in an elastic artery having porosity effect. In the initial stage we will concentrate on simple oscillatory function containing a single harmonics following Dutta and Tarbell (1996)

$$\frac{dp(t)}{dt} = K + m \cos mt \quad \text{.....(18)}$$

$$R(t) = \bar{R}(1 + k \cos(mt - \theta)) \quad \text{.....(19)}$$

Where \bar{R} and \bar{k} the mean parameters m and K are the amplitude parameters, θ is the phase angle and w is the frequency.

$$Q(t) = Q(1 + K_0 \cos(mt - \theta))$$

$$T_m(t) = r(1 + k \cos(mt - \theta))$$

where $Q(1)$ and (b) are the low rate and wall shear stress respectively. For our simulation, the amplitude of second harmonics are always less

than 8% of that of the first harmonics, indicating very small distortion. For physiological flow simulation, we have used multi-harmonic physiological flow waveforms

$$z = P/Q \quad \text{and} \quad z = P/R$$

Where z is the impedance and z_w is the well impedance.

4. Numerical Method

The above occurring equation is solved by using Finite Difference method by taking appropriate grid scaling.

$$\frac{\omega_{i+1}^{n+1} - \omega_i^n}{\Delta t} = \frac{1}{P_0^n} \frac{P_{i+1}^{n+1} - P_i^n}{\Delta z} - \frac{1}{e_0^n R_0^n} \frac{\tau_{i+1} - \tau_i}{\Delta s} + \left[\frac{s}{R_0^n} \frac{R_{i+1}^{n+1} - R_0^n}{\Delta t} \right] - \frac{4i^n \omega_i^n - \omega_{i-1}^n}{4} + \frac{4i^n \omega_{i+1}^n - \omega_i^n}{4} - \frac{y_i^n}{\Delta s} + \frac{R_0^n}{4} \left(\frac{i+1}{\Delta s} - \frac{4i}{s^n} + \frac{4i^n}{s^n} \right) - \frac{-6}{e_0^n} B^2 \omega_i^n - \frac{q}{k} \omega_i^n$$

$$\text{Where} \quad \frac{dv}{dt} = \frac{v_{i+1}^{n+1} - v_i^n}{\Delta t}, \quad \frac{dv}{dx} = \frac{v_{i+1}^{n+1}}{\Delta x} + 0(\Delta x)^2 \quad \text{.....(22)}$$

$$\left(\frac{\partial w}{\partial \xi} \right)_i = \frac{w_i^{n+1} - w_i^n}{\Delta \xi} + o(\Delta \xi)^2 = 0. \quad \text{at } \xi = 0$$

$$w = 0. \quad u = \left(\frac{\partial R}{\partial t} \right)_i = \frac{R_i^{n+1} - R_i^n}{\Delta t} \quad \text{at } \xi = 1$$

This approach is applied for temporary discretization to ensure a second-order accurate solution both in time and space for both convection and diffusion terms. The axial velocity is computed solving momentum equation (22) given by suitable constitute equations and the radial velocity is calculated from equation (12). 1 pressure gradient parameters law are chosen initially so that flow wave forms produced for Newtonian fluid in the rigid tube would have an amplitude approximately equal to the mean (4,-1) and mean flow rate characteristic of the theoretic aorta under normal values $Q=7.7L/min$, a 12. m 18 dyne/cm², and $B=2$. The wall impedance has been modeled as a purely elastic element with zero phase angle and constant modulus, including taking porous medium (OR). The effect of porosity is(21)

5. Numerical Results and Discussion

After performing the computation by the finite difference method. we have concluded that by the

effects of magnetic field velocity of blood flow through Arteries decreases because of scattering of blood constituents and Porosity also act a major role of decrease in velocity and Pressure Arterial flows as we have developed in our model which

are shown graphically to understand the result easily. We hope our study will be useful for medical scientists to diagnose various heart diseases.

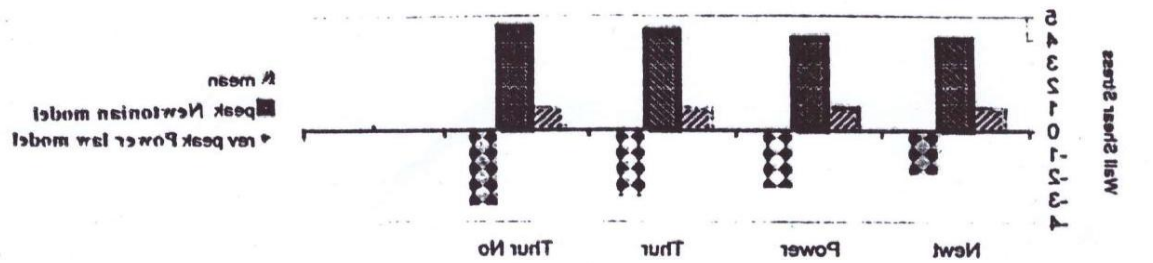


Fig.1: Comparison of wall shear stress for different rheological models in a rigid artery with a fixed pressure gradient μ in the

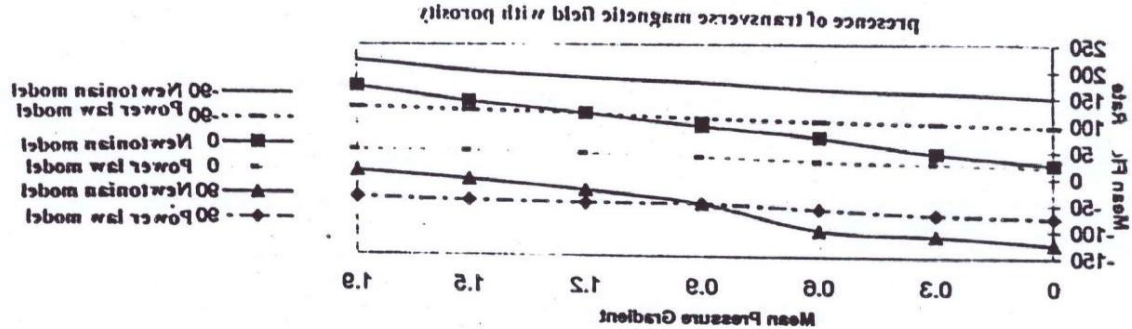


Fig.2: Mean flow rate versus mean pressure gradient in an elastic artery for different parameters, $R=1.9$ cm, $B=3$ ($-30,0,90$) denote the impedance phase angle

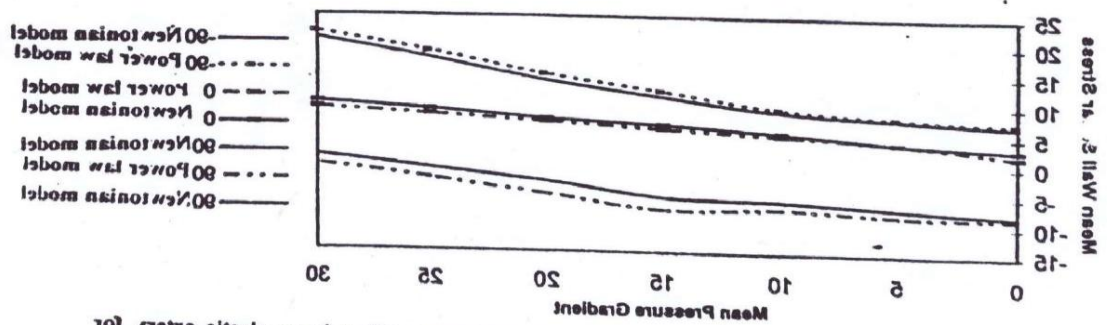


Fig.3: Mean Wall Shear Stress versus mean pressure gradient in an elastic artery for different parameters, ($-30,0,90$) denotes the impedance angle, $k=18$ dyne/cm², $R=1.9$ cm, $B=3$

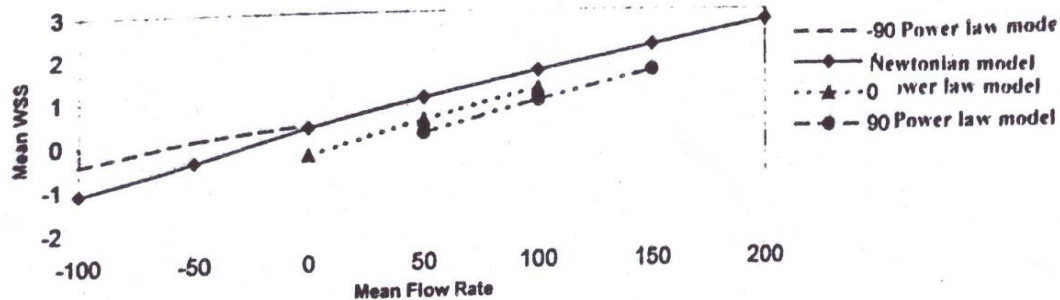


Fig.4: Mean wall shear stress versus mean flow rate for the Power law model for different parameters. (-90,0,90) denotes the impedance angle. $k=18\text{dynes/cm}^3$, $R=1.4$, $B=2$.

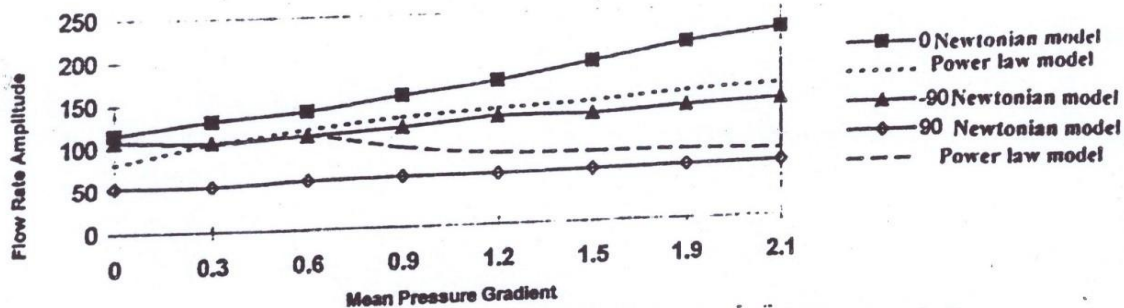


Fig.5: Flow rate amplitude versus mean pressure gradient in an elastic artery in the presence of transverse magnetic field with porosity, $k=18\text{dynes/cm}^3$, $R=1.4$, $B=2$

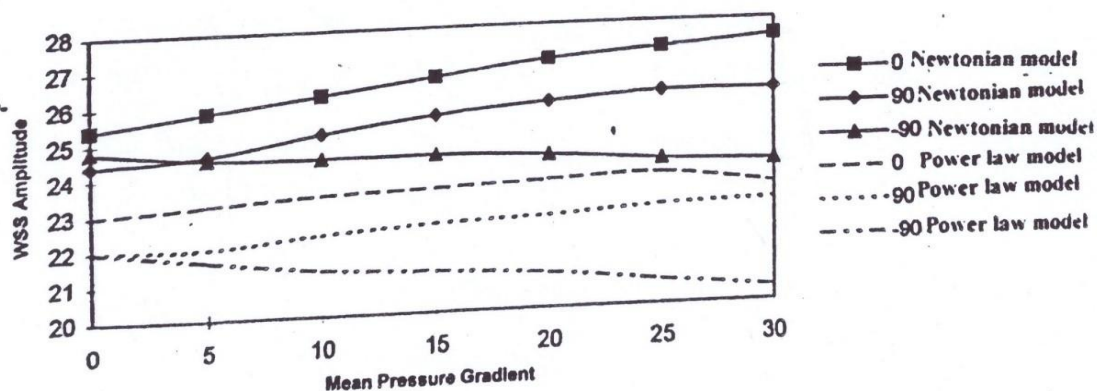


Fig.6: Wall Shear Stress amplitudes versus mean pressure gradient in an elastic artery in the presence of magnetic field with porosity
 $k=40.5\text{dynes/cm}^3$, $R=1.4$, $B=2$

References

1. Anderson, J.L. and Malone DM (1974) Mechanism of osmotic flow in porous membrane. *Journal of biophysical*, vol.14. pp.957-982
2. O-Brein, V. and Ehrlich, L.W. (1985) simple pulsatile flow in artery with a constriction. *Journal of Bio-Mechanics*, Vol.18. pp.117-127.
3. Mishra, J.C. and Chakravarty, S. (1986) Flow in arteries in the presence of stenosis. *Journal of Bio-Mechanics*, Vol.19. pp. 13-20.
4. Stefan, W. (1986) Flow in porous media: I A Theoretical deviation of Darcy law. *Journal of Transport media*, Vol. 1. pp. 3-25.
5. Stefan, B. Cuthiel, D. (1990) Effects of core scale heterogeneity on steady state transient fluid flow in porous media: Numerical Analysis, *Journal of Water Resources Research*. Vol.26. No.5 pp. 863-874.
6. Bachu, S. and Cuthiel, D. (1990) Effects of core scale heterogeneity on steady state and transient fluid flow in porous media: Numerical analysis, *Journal of Water Resources Research*, vol.26. No.5 pp.863-874.
7. Amiri, A. and Vafai, K. (1994) Analysis of dispersion effects and non thermal equilibrium, non Darcian, variable porosity, Incompressible-flow through porous media, *International Journal of Heat and Mass Transfer*. Vol.37. pp. 939-954.
8. Simurek, J. and Donald, L.S. (1994) Two dimensional transport model for variably saturated porous media with major ion chemistry, *Journal of Water* pp.1115-1133. resources, vol.30.
9. Bruce, B.D. and Peter, K.K. (1996) Macro Transport of a biological reacting solute through porous media, *Journal of Bioresearch*. Vol.32. pp. 307-320.
10. Zou, X.Y and Pereira J.C.J (1997) Numerical study of combustion and pollutants formation in inert non homogeneous porous media, *Cmbust. Sci and Tech*. 130. pp. 335-364.
11. Teeluck, M. and Shutton, B.G. (1998) An extracted porous pipe made up of recycled automobile tyres tested to study its efficacy for use as a micro Irrigation lateral, *Journal of Agricultural Water Management*. Vol.38. pp. 123-134
12. Malico, I. and Pereira, J.C.F. (1999) Numerical study on the influence of radioactive properties in porous media combustion, *ASME Journal of heat Transfer*. 123, pp. 951-959
13. Zhangxin, C., Guan, Q. and Richard, E.E (1999) Analysis of a compositional model for fluid flow in porous media, *SAMI. AFFI. Math*. vol. 3 pp. 747-777
14. Bouhouch, m. Part, M. and Bisson, H. (1999) variable density flow in porous media: A study by means of pore level numerical simulation, *International journal for Numerical methods in Fluids*, Vol.30. Issue 6 pp. 725-742.
15. Brenner, G. Pikenacker, K. Pikenacker, O. Tirmis, D. Wawrzinek, K. and Weber, T. (2000) Numerical and Experimental Investigation of matrix-established Methane/Air combustion in porous media, *Combust Flame*. vol.123. Pp. 201-213.
16. Karner, G. and Perktold, K. (2000) Effect of endothelial injury and in increased blood pressure on accumulation in the arterial wall: A numerical Study, *album Journal of Bio-mechanics*. Vol.33. pp. 443-4555
17. Marafie, A. and Vafai, K. (2001) Analysis of non-Darcian effects on temperature differentials in porous media, *International Journal of Heat and mass Transfer*. Vol.44. pp. 4401-4411.
18. Kendall, T., Harris, A., Sheikh, H. and Nnane, A. (2001) Phase change phenomena in porous media-A non localthermal Equilibrium model, *International Journal of Heat and Mass Transfer*, Vol. 44. pp. 1619-1625.
19. Alazami, B. and Vafai, K. (2001) Analysis of fluid flow and heat transfer interfacial condition between porous medium and fluid layer, *International Journal of Heat Mass Transfer*. Vol.44. pp.1735-1749,
20. Oswald, J.G and Baraki, M. (2001) Migration of colloids in discretely fractured porous media: Effect of colloidal matrix diffusion, *Journal of contaminHydrol*. vol. 52. pp. 213-244
21. Sharma, G.C, Jain, M. and Kumar, A. (2001) Finite Element Galerkin approach for a computational study of arterial Flow, *Journal of Applied Mathematics and Mechanics*. vol.22 (9) pp. 1012-1118.
22. Shih, T.C., Kou, H.S. and Lin, W.L (2002) Effect of effective tissue conductivity on thermal dose distribution of living tissues with directional

blood flow during thermal therapy. *Int. Common. Heat Mass Transfer*. vol. 29. pp.115-126

23. Hsisshih, W., Cheng, P. and K. Chen Chao (2003) Non uniform porosity and thermal dispersion effects on natural convection about a heated horizontal cylinder in an enclosed porous medium, *International Journal of Heat and Mass Transfer*. vol. 35. pp. 3407-3418.

24. Laurant, O., Golfier, F., Michel, A.B. and Michel, Q. (2004) A two equation model of Biologically reactive solute transport in porous media, *International Conference on Water resources CMWR 2d. J. Carrera (Ed) CIMNE Baranall*

25. Nicolson, D. and Petropoulos, J.H.(2006) Capillary models for porous media; IV Flow properties of parallel and serial capillary models with various radius distribution, *Journal of Physics*. Vol.6. pp. 1737-1748

26. Sanyal D.C, Das, K. and Debnath, S. (2006) Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration, *Journal of Physics. Sci.* vol. 11 pp.43-56.

27. Raddaish, P. and Prasada, D.R.V. (2007) Finite Element analysis of convective flow through porous medium in a rectangular duct, *Bulletin of Pure and Applied Science-Mathematics*, jan-01, 2008

28. Arthur, J.K, Douglas, W. R. and Tachie. (2008) A PIV study of fluid flow through parallel communicating layers of porous media, *International Symposium of the Society of core analysis held in Abu Dhabi UAE 29 October to November-2008*.

29. Nakayama, A. and Kuwahara, F. (2008) A general bio heat transfer model based on theory of porous media. *International Journal of Heat and Mass Transfer*. Vol.51. pp.3190-3199.

30. Willitiam, T. W. Werth, C.J and Valochi, A.J (2008) Evaluation of the effects of porous media structure on mixing controlled reaction using pore scale modeling and micro model experiments, *Journal of Environmental Science and Technology*, vol. 42(2) pp.3185-3193.

31. Esmaeilzadeh, F., Zeighami, M.E. and Fathi, J.(2008) 1-D Modeling of Hydrate decomposition

in porous media *International Journal of Chemical and Biological Engineering*. Vol.4. pp.324-332.

32. James, K.A., Douglas, W. R. and Tachie, M.F. (2008) A PIV study of fluid flow through parallel communicating layers of porous media. *International Symposium of the society core analysis held in abu held in Abu Dhabi, UAE, 29 Oct to 02Nov. 2008*.

33. Rathod V.P. and Tanveer, S. (2009) Pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic fields, *Bull, Malays Math Sci, Soc.* vol.32. pp. 245-259.

34. Pedro, A., Thomas, B. Rosin, R. and Houle, J. (2009) Modeling urban stress quality treatment: Model development and application to a surface sand filter.

Journal of Environmental Engineering. vol.136. pp.1943-1953.

35. Laurant, O., Golfier, F., Michel, A.B. and Michel, Q (2010) A two equation model of biologically reactive solute transport in porous media. *International conference on water resources CMWR 2d.J.Carrera(Ed)CIMNE Barreonal*.

36. Mansourish, N. and Shoki, N. (2011) Non linear effects of salt concentrations on evaporation from porous media. *Geo physical Research Letter:doi 10, 1029/2011GL.050763*.

37. Sigh, J. and Rathee, R. (2011) Analysis of non-Newtonian Blood flow through stenosed vessels in porous medium under the effect of magnetic field. *International Journal of the Physical Sciences*. Vol.6 (10) pp.2497-2502.

38. Bidi, M. Saffaraval, M. and Heyrani, N. (2011) A two dimensional study of combustion in porous media, *Euro Therm seminar heat transfers in porous media. Ecoledes Mines france June 4-6. 2011*

39. Khaled, A.R.A and Vafai, (2012) The role of porous Media in modeling flows heat transfer in biological tissues, *International Journal of Heat and Mass Transfer*. Vol. 46.pp. 4989-5003.

40. Shah, R.S. (2012) Response of blood flow through an atherosclerotic artery in the presence of magnetic field

41. Dabiri JO, Koehl MAR (2013) "Simultaneous field measurements of ostracod swimming behavior and background flow," *Limnology and*

Oceanography: Fluids and Environment 1: 135-146.

42. Katija K, Colin SP, Costello JH, Dabiri JO (2014) "Quantitatively measuring in situ flows using a self-contained underwater velocimetry apparatus (SCUVA), Journal of Visualized Experiments 56: e2615

43. Ruiz LA, Whittlesey RW, Dabiri JO (2015) "Vortex-enhanced propulsion," Journal of Fluid Mechanics 668: 5-32.

44. Colin SP, Costello JH, Hansson U, Titelman J, Dabiri JO (2016) "Stealth predation: the basis of the ecological success of the invasive ctenophore Mnemiopsis leidyi." Proceedings of the National Academy of Sciences of the USA 107 (40): 17223-17227.

45. Whittlesey RW, Liska SC, Dabiri JO (2017) "Fish schooling as a basis for vertical-axis wind turbine farm design," Bioinspiration and Biomimetics 5: 035005.

46. Breitburg DL, Crump BC, Dabiri JO, Gallegos CL (2018) "Ecosystem engineers in the pelagic realm: Habitat alteration by species ranging from microbes to jellyfish," Integrative and Comparative Biology 50 (2): 188-200.

47. Nawroth JC, Feitl KE, Colin SP, Costello JH, Dabiri JO (2019) "Phenotypic plasticity in juvenile jellyfish medusae facilitates effective animal-fluid interaction." Biology Letters 6 (3): 389-393.

48. Dabiri JO, Colin SP, Katija K, Costello JH (2020) "A wake-based correlate of swimming performance and foraging behavior in seven co-occurring jellyfish species," Journal of Experimental Biology 213 (8): 1217-1225.

49. O'Farrell C, Dabiri JO (2021) "A Lagrangian approach to identifying vortex pinch-off," Chaos 20: 017513.

50. Dabiri JO, Colin SP, Katija K, Costello JH (2022) "A wake-based correlate of swimming performance and foraging behavior in seven co-occurring jellyfish species," Journal of Experimental Biology 213 (8): 1217-1225.