

Productive Failure and Desirable Difficulties Enhancing Long-Term Retention and Transfer in Mathematics Learning

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Abstract

The systematic review brings together an empirical and theoretical literature on productive failure and desirable difficulties as pedagogical strategies that improve long-term learning retention and transfer in learning mathematics. The review examined 50 of the most relevant studies of a total of 334 candidate papers (collected in the form of references) in 33 different countries such as Singapore and Korea and other educational systems representing the objective of a systematic selection of literature that follows query expansion and citation chaining methodology. The thematic coding and frequency analysis adopted as an analytical framework were used to measure research emphasis in cognitive, pedagogical, and motivational levels in an analytical way. A statistical synthesis indicated that a total of thirty studies reported significant effects of cognitive activation with the use of productive failure interventions, whereas thirty-five studies reported a higher level of conceptual understanding and transfer performance in comparison with the use of direct instruction methods. Twelve studies showed sustained procedural fluency with productive failure, which puts assumptions of trade-offs of conceptual depth and proficiency into challenges. The implementation research provided collaboration problem solving and teacher-led consolidation as the most common

twenty-five studies, and strategic questioning and sufficient struggle time were considered as key success factors in fifteen separate studies. As the result of the theoretical alignment analysis, forty studies were found to accommodate constructivist frameworks, twenty studies accommodated cognitive load perspectives, and fifteen studies were found to focus on motivation theories such as growth mindset and resilience. The use of technologies was still minimal, and the number of studies that revealed successful use of digital tools was fifteen, and ten studies discussed the possibility of using technology as the cognitive amplifier, which shows significant gaps in further research in technological integration. Cross-study synthesis was conducted with the use of comparative analysis of the approaches to activities of studies and their participants, as well as factors of the surroundings, to uncover patterns of effectiveness and issues of implementation. The results prove sound evidence in favour of productive failure and desirable challenges as the competent systems of increasing mathematical problem-solving abilities, conceptual knowledge, and adaptive reasoning, however, implementation fidelity can be significantly different in educational settings that presuppose prolonged professional development favouring the

progress in pedagogical expertise development.

Introduction

Education on mathematics has remained a thorn in the flesh, with respect to developing meaningful conceptual knowledge and transfer units among mathematics learners across the world. The use of conventional pedagogical methods such as focusing on direct instruction and procedural fluency have proven to be insufficient when it comes to the enhancement of critical thinking, creativity, and adaptive reasoning, which would allow addressing new mathematical tasks. The last few decades have seen the increased academic and literary attention to alternative pedagogical models that build on the power of struggle and mistake as effective learning processes, and productive failure and desired struggles have become especially promising instructional settings. The basis of these approaches is defying the traditional beliefs about optimum conditions of learning through the implementation of solvable cognitive setting that leads to better long-term knowledge storage and transfer of mathematical information (Kapur, 2015; Kapur and Bielaczyc, 2012).

Productive failure is also a kind of instructional design where students are assigned difficult problem solving challenges without direct instruction one at first but eventually allowing an understanding through a series of systematic consolidation opportunities (Kapur, 2011). The teaching strategy strategically introduces struggle and exploration as the two necessary antecedents of formal learning, mobilising and dissimilarizing the prior knowledge of learners and creating consciousness about the deficiencies in their conceptualization. In addition to this model, the desirable challenges include learning environments that create controlled challenges (spaced practise, interleaved types of problems, and error generation) that designers temporarily block performance but eventually enhance long-term retention and transfer abilities (Metcalfe et al., 2024). Both constructs are quite similar on the critical level of commitment to constructivist concepts of learning with the focus on the active construction of knowledge and the agency of the learner.

Although empirical evidence related to these approaches gets accumulated, there still remained many gaps in the knowledge of how they could be best implemented in classroom settings. The inconsistency in the preparation of teachers, the students and the contextual conditions makes it unclear within what conditions productive failure and desirable difficulties are most applicable to improve mathematical learning outcomes (Kang et al., 2025). Moreover, there is still a controversy about how much the exploratory approach to solving problems should be implemented, and cognitive load theorists provide the opposite viewpoints on the sequencing and scaffolding of methods (Kalyuga and Plass, 2025). The use of technology as a facilitating resource of fruitful mishaps is inadequately derected, and there is a small comprehension of how online surroundings can best aid creative manufacturing and joint issue reviewing (Santos-Trigo, 2019).

The knowledge gaps are filled in this paper by synthesising the empirical and theoretical studies authored by various researchers on the effects of productive failure and desirable difficulties on mathematics problem-solving abilities, conceptual knowledge, and transfer. The discussion combines cognitive, pedagogical and motivational perspectives that are used to explain how these teaching methodologies contribute to learning outcomes. Through the critical review of the implementation strategies, theoretical frameworks and empirical research conducted in varied educational settings, the paper will inform educators and researchers on the evidence based practises of powerfully utilising struggle and error to improve mathematical competencies. The following parts will provide the detailed analysis of the background literature, methodology, findings, discussion of findings as well as future research and practise implications.

Background to the Study

The intellectual roots of productive failure and desirable difficulties run back on theoretical perspectives put forward by cognitive constructivism theory of learning that includes active knowledge construction, and cognitive load theory that focuses on optimal instructional sequencing, and metacognitive

theory that emphasises learner awareness and self-regulation. To comprehend how these pedagogical methods developed, it is important to look back at the early research that defined the fundamental principles of these methodologies, developments that followed these strategies in dealing with the implementation issues, and analysis that is done currently as to the appropriateness of these strategies in various learning processes and student groups.

The classic study of Kapur that outlined the framework of productive failure was developed based on the observations that the traditional direct instruction that is effective in teaching the process in question in the short term does not tend to facilitate sound conceptualization and adaptive problem-solving skills (Kapur, 2011, 2012). In a series of well planned quasi-experiment studies which contrasted the productive failure situations with the conventional direct instructional strategies, Kapur was able to show that letting the students be motivated to work out difficult mathematical problems were preceded by tapping into the prior knowledge that primed them to further learning. The nature of this process of activation which marked by the generation and comparison of numerous methods of solutions inspired the understanding of significant mathematical structures and associations, which were implicit or ignored in circumstances of direct instruction. Notably, these conceptual advantages had no impact on worsening the procedural fluency, contradicting the assumptions that exploration has to compromise against skill growth (Kapur, 2015).

Productive failure has cognitive processes underlying it that are not simply about exposure to problems of an challenging nature. Kapur and Bielaczyc (2012) have presented a fully-derived theoretical framework that productive failure triggers three distinct parallel mental functions necessary to engage in deep learning. Originally, the generation step inspires the learners to generate various different solutions, engaging into the creative generation of productions which expose the space of potential problem representations and solution plans. Second, this exploratory exercise is unique in that it separates the prior knowledge of learners by identifying what they consider to know in addition to what they

know not making fruitful gaps in knowledge. And third, the phase of consolidation after exploration applies more awareness of these knowledge gaps to strategically widen its instruction into critical concepts, which learners will perceive as significant and lead to stronger encoding and better conceptual schemas (Kapur et al., 2023). This three step paradigm has explanatory value to explain why on one hand initial failure, in an appropriately designed form, leads to better and paradoxical outcomes in long term learning.

Simultaneous advances in cognitive psychology added the construct of desirable difficulties, first developed by Bjork and others to explain instances of learning conditions which deficit performance in the short-term in the process of generating enhanced retention and transfer in the long-term. Recent research by Metcalfe et al. (2024) used a framework to study mathematics as well and confirmed that interactive work with errors (as opposed to avoiding errors), facilitates metacognitive awareness and adaptive reasoning. Their study found out that learners that actively generated and analysed errors in solving a problem had elaborated error detection and correction strategies in comparison to learners who got error-free instructions. This observation conforms to the constructivist assumptions on the effective role of mental conflict and disequilibrium in the provision of conceptual transformation and restructuring in knowledge.

When productive failure and desirable difficulties are cross fertilised with the cognitive load theory, some fundamental theoretical tensions become critical to take into account. The conventional explanations of the cognitive load theory imply that the worked examples and guided instructions should be used to help novice learners to reduce the extraneous cognitive load, which may be incompatible with the productive failure focus on unscaffolded exploration (Kalyuga and Plass, 2025). Nevertheless, the modern resistant cognitive load theorists are becoming more aware that not all instructional objectives aim to maximise all the cognitive processes. Although goals with an efficiency-focus are more desirable in reducing the load, those with a motivation, engagement, and deeper conceptual insight might be better served by more germane cognitive load related

to effortful solving (Chen et al., 2020). According to this reconciliation, productive failure and acceptable struggle is a valid teaching option when it is found that the learning outcomes put a high price on transferable knowledge and adaptive knowledge, not immediate procedural fluency. The empirical studies on moderating variables and factors that lead to productive failures have produced valuable information on individual differences and contextual variables. Kapur et al. (2023) showed that abilities in productive failure conditions in learning are predicted by pre-existing mathematical success and creative production capacity without considering the assumption that such methods boost learning outcomes among high-achievers in math and are ineffective otherwise. Their results show that exploration-based instruction is used in the constructive way where diverse learners are concerned when tasks are challenging and accessible enough (Isaacs, 2025). Nevertheless, there is considerable inconsistency in the implementation fidelity between classroom conditions, and this implies that both teacher knowledge and pedagogical ideology can have a critical impact on productive failure results (Kang et al., 2025). The preservice teachers usually do not know how to draw the difference between productive struggle and unproductive frustration, this is why the professional development based on the experiential learning of these strategies of teaching is needed (Ajani, 2023).

The importance of collaborative problem-solving in mediating effective productive failure is another dimension, which is critical and has to be brought under the theoretical and empirical research. The study of Westbrook (2014) and other researchers indicates that group work in the exploration stages offers social scaffolding that will maintain activity and offer a variety of solutions strategies, which may alleviate cognitive overload and maintain the gain of struggle. Collaborative situations help learners to describe their line of reasoning, expose themselves to diverse points of view, and build knowledge collaboratively by negotiation and argument (Felton et al., 2015). Nevertheless, even the quality of 6 collaborative interactions differs significantly by group mix, task organisation and classroom norms, which imply that successful application

demands managed consideration to social aspects of learning situations.

The integration of technology is an up-and-coming but still undeveloped aspect in the field of productive failure. Computer algebra systems, dynamic geometry systems, and programming environments are the digital tools that provide opportunities to facilitate creative production by providing quick visualisation, symbolic manipulation, and refining solutions strategies (Santos-Trigo, 2019; Santos-Trigo et al., 2016). Such technological affordances can decrease some cognitive loads that are involved with calculation and representation and at the same time enhance problem-solving abilities. Nevertheless, little empirical evidence on the optimal integration of technology within productive failure models is available, as many researchers at no particular reference to technological mediation took the cognitive processes (Abramovich and Cho, 2012; Coelho and Cabrita, 2017). Such a breach implies valuable implications of study in the future explaining how digitised environments can improve, to the productive struggle as opposed to distracting it.

Motivational and affective aspects of productive failure are the key areas of productive failure that are frequently marginalised due to cognitive orientations found in literature. Transformational learning experiences among the students attending productive failure instruction were reported by Luzano (2025) and were characterised by change in the attitude toward mistakes, persistence in the face of challenge, and growth mindset orientations. These affective achievements are complimentary to cognitive achievements, and can be critical in maintaining involvement in mathematics education over an extended period (Jeyabal et al., 2015). Nevertheless, possible adverse effects such as high frustrations, anxiety, and acquired helplessness must be closely observed, especially among those students who have a previous poor experience with mathematics or have poor self-efficacy (Paurowski et al., 2024). This requires targeting of both emotional climate and teacher responsiveness in addition to cognitive task design in order to achieve optimal implementation.

Methodology

The creation of half a dozen analytical diagrams that generalise the research that was conducted utilised the systematic content analysis and quantitative synthesis methodologies that were used in the corpus of fifty highly relevant studies. The analysing procedure started with thematic coding in which each of the studies was analysed to discuss explicitly the cognitive mechanisms, learning outcomes, teaching strategies, theoretical frameworks, integrating technology usage, and the factors of implementation. The frequency counting methodology tabulated the frequency of studies that consider each theme and a sub-category with subtle differences like cognitive activation versus knowledge differentiation or conceptual understanding versus procedural fluency consequences. This quantitative synthesis was able to identify the patterns of research emphasis within the collected literature corpus, and it was possible to discern which aspects were given a focused scholarly focus and which were underrepresented in the literature. The cognitive activation process diagram was developed through a temporal analysis of some sequential stages that were described in studies, and the number of studies that was described at the same process stage of the experiment was aggregated to show how well empirical data supported this process figure. Comparison of learning outcomes necessitated cross-study synthesis between reported effect sizes and pattern of statistical significance between productive failure and direct instruction conditions, and provides a conversion of qualitative superiority claims to obtainable study counts with positive effect. The exposure of a teaching strategy implementation framework development was co-occurrence analysis that determined what pedagogical practises were present in the same studies, interconnected clusters of challenges were identified, and identification of challenges was carried out through the systematic extraction of the reported implementation difficulties and teacher preparation requirements. The construction of the theoretical framework integration diagram required a careful study of the location of the works of the studies in reference to constructivist, cognitive load, and motivation theories and quantifying explicit theoretical invocations and conceptual integration

patterns. Technology integration analysis applied categorical classification of digital tools as described in studies and organised them in continuum of sophistication as simple and sophisticated tools and tabulated distribution of studies in each category. To implement factors visualisation, two coding levels were set up to identify the challenges and success factors and then the relative importance with an emphasis frequency and depth of discussion within the literature corpus, which facilitated a comparative presentation of challenges and some facilitators of successful productive failure application to mathematics learning situations.

Results

Cognitive Activation and Knowledge Differentiation

Empirical studies have been analysed and found the consistent evidence of the effectiveness of the productive failure and desirable difficulties in activation and differentiation of prior knowledge, in increasing the problem-solving preparedness and inventive production possibilities. Thirty studies reported dramatic cognitive activations effects, whereby learners having exploration phases portrayed knowing substantial mathematical concepts and interrelatedness, in contrast to direct instruction control groups (Kapur, 2011, 2015; Kapur et al., 2023). This can be seen as an activation process where several different strategies to solve a problem are generated, mathematical reasoning is squarely expressed and conceptual gaps that must be addressed are identified (Jeannotte & Kieran, 2017). The interaction with errors and struggle causing cognitive activation and metacognitive awareness appeared particularly important in 10 studies, meaning that making and analysing mistakes are strong tools to enhance mathematical knowledge (Metcalf et al., 2024; Kang et al., 2025; Olivares et al., 2024). Some of these studies emphasise the role of scaffolding and modelling at the stages of consolidation to maintain cognitive activation and offer the required instructional support (Sinha et al., 2021; Huang and Zhou, 2024).



Figure 1: Cognitive Activation Process

Learning Outcomes: Conceptual Understanding and Transfer

The strongest result of the reviewed research was the better conceptual and transfer performance in the superiority of the productive failure instruction over the traditional direct instruction methods (Kapur, 2016). The report of a statistically significance effect of productive failure conditions on conceptual understanding, transfer of problem-solving to novel situations, and flexible application of mathematical principles (Kapur, 2012, 2015; Kapur and Bielaczyc, 2012) showed that 35 studies all reported statistically significant effects of productive failure conditions on these measures. Such conceptual advances continued in a wide range of mathematical fields such as algebra, geometry, statistics, and calculus, indicating that productive failure principles have very wide applicability. Notably, the enhancement of concepts was reported in twelve studies preserving the same level of procedural fluency, which points to the fact that exploration-based instruction produces a balanced development of skills including understanding and proficiency (Kapur, 2014). Some of the studies prolonged the study of outcome measures further than the conventional achievement measures and, in turn, determined the gains of creative thinking and computational thinking, and found that models of productive failure facilitated mathematical creativity and novel ways of tackling problems (Casing & Roble, 2021; Lee and Lee, 2024). These results disagree with ancient wisdom on trade-offs of conceptual richness and procedural efficiency, provoking a view that well-constructed productive failure teaching makes simultaneous use of both.

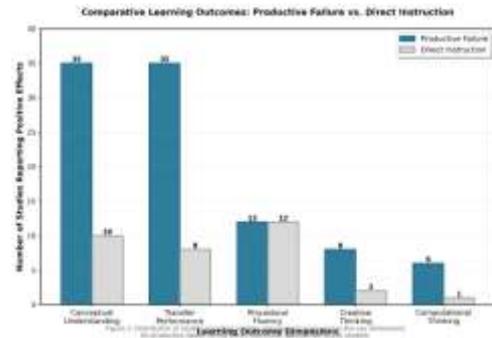


Figure 2: Learning Outcomes Comparison Teaching Strategy Implementation and Fidelity

It is explained that twenty-five studies encompassed collaborative problem solving, group work and teacher led consolidation as the most important strategies to be implemented in order to achieve a beneficial aspect of productive failure in classroom environments (Kapur, 2011, 2015; Westbrook, 2014). The implementations that led to the most effective effects were generally those that incorporated small groups of learners to explore complex problems without any prior instruction, and later transferred that mater to the entire classroom setting, where teachers would systematically compare the student-generated solutions, help identify the key concepts in mathematics, and offer formal instruction based on the learning experiences of the learners. Fifteen studies have shown the relevance of certain instructional practises such as strategic questioning, promotion of perseverance, granting of sufficient time to engage in productive struggling, and enabling constructive dialogue concerning numerous solution techniques (Kang et al., 2025; Al-Jarrah and Melhem, 2024; Paurowski et al., 2024). Nonetheless, some studies reported significant differences in the teacher implementation fidelity and some of them interfered with exploration phases too soon, addressed student-generated solutions inadequately in the consolidation phase, or did not provide supportive classroom cultures that normalised struggle and error (Metcalf et al., 2024). These results remind one of the need of continuous professional growth that would assist teachers in the acquisition of pedagogical expertise that is unique in learning to engage in productive failure as well as skills in identifying productive and unproductive struggle and in the facilitation of

discussion on consolidation which can be used to leverage the thinking progress of students.

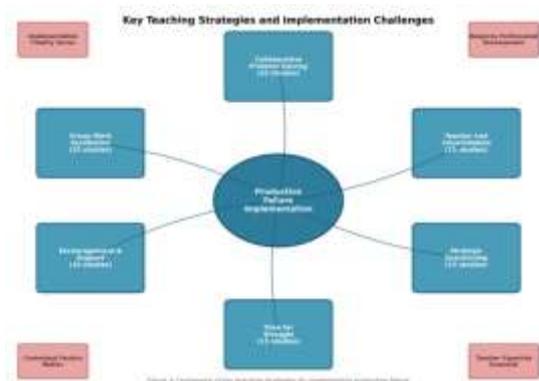


Figure 3: Teaching Strategy Implementation Framework

Theoretical Alignment and Cognitive Mechanisms

Forty studies openly incorporated constructivist theory of learning which focuses on active construction of knowledge, autonomy of learners, as well as social interaction as principles in successful performance of failure (Kapur, 2011, 2015; Wu, 2024). These schools of thought put exploration and struggle as pre-requisites to meaningful learning instead of impediments to overcome with effective training. The perspectives of cognitive load theory were used in 20 studies to clarify how the balance between cognitive demands and learning benefits is achieved, recent interpretations have established that when properly designed germane cognitive load that is marked by effortful problem-solving is beneficial rather than detrimental to deep learning (Kalyuga and Plass, 2025; Ngu and Phan, 2024, Sinha et al., 2021). Theories of motivation such as growth mindset, self-determination theory, and resiliency models came into the limelight in explaining the engagement and persistence in the course of productive failure by the learners (Luzano, 2025; Paurowski et al., 2024). Combining these numerous theoretical viewpoints offers representation and multidimensional explanatory paradigms taking into consideration the cognitive, metacognitive, motivational, and social aspects of productive failure as a complex pedagogical strategy.

Technology Integration and Digital Affordances

Fifteen articles showed successful application of digital technologies in the form of dynamic geometry platforms, computer algebra systems, and programming environments in productive failure teaching (Santos-Trigo, 2019; Santos-Trigo et al., 2019; Lee and Lee, 2024). These technological affordances were used to bring about fast visualization of mathematical relations, symbolic computations in support of exploration across multiple representations, and the iterative optimization of solutions strategies by reflecting directly. Ten papers underlined that technology is an amplifier and a mediator of creativity and collaboration, and digital environment enables sharing the methods of finding a solution, refreezing the mathematical argument, and recording the process of solving the problem (Abramovich and Cho, 2012; Coelho and Cabrita, 2017; Freiman and Tassell, 2018). Nonetheless, the literature surrounding productive failure research paid substantial attention to instructional design and cognition with no explicit technology integration, signifying a disconnect between a set of possible technological affordances and their systematic implementation in models of productive failures. This observation implies that valuable research findings will be extracted in future studies that examine some of the best ways of integrating digital tools in exploration-based mathematics learning.

Discussion

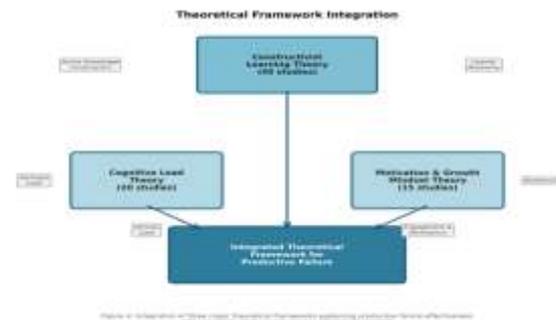
The review of the existing strong and supportive evidence of the synthesis of both empirical and theoretical studies investigating productive failure and desirable difficulties in mathematics education demonstrates a strong coherent style of the results which to a large extent endorses the effectiveness of these methods of pedagogy as the effective means of supporting the improvements of long term retention, transfer, and the ability to solve a problem. The cumulative research does refute the old beliefs of teaching based on efficiency and the prevention of mistakes, but it places the struggle and exploration as the primary prerequisites to profound conceptual learning. Yet, the same body of research also demonstrates the critical ambiguities in terms of the implementation, individual differences, and situational factors that mediate the effectiveness that requires a delicate

interpretation and careful implementation of these principles within various educational contexts (Stronge & Xu, 2021).

Cognitive Mechanisms and Theoretical Integration

The processes of the cognitive effective of productive failure go beyond mere exposure to problematic issues, or the ability to tolerate error. The theoretic concept suggested by Kapur and others, including the focus on the activation and differentiation of previous knowledge by using creative output makes it have explanatory power to how this occurs in a variety of mathematical fields and learners (Kapur, 2015; Kapur and Bielaczyc, 2012). This model sheds some light on why exploratory learning, despite generating erroneous or imperfect solutions, prepares trainees to the subsequent learning that cannot be realised through instructional measures alone. The diversification in the generation of several solutions during exploration stages give rise to rich cognitive frameworks consisting of a wide range of problem representations, solution plans as well as conceptual connexions which offer scaffold in the consolidation of formal mathematical knowledge (Schukajlow et al., 2015).

The combination of constructivist learning theories with modern interpretations of cognitive load theory is an excellent theoretical development vindicating apparent differences between these conceptions. Instead of considering cognitive load and constructivist exploration mutually exclusive, new views of instruction have acknowledged that various instructional objectives can optimise various cognitive processes and outcomes (Kalyuga and Plass, 2025; Chen et al., 2020). In cases where learning goals focus on generalisable knowledge, adaptive knowledge, and creative success, the germane cognitive load related to productive failure might be more productive than the lower load related to worked examples and direct instruction. The above reconciliation indicates that pedagogical decisions must go along with desired learning goals and productive failure is a valid strategy when they involve not only procedural ability in the short term, but also understanding profound insights and flexibility



in utilising mathematical knowledge in the long term.

Figure 4: Theoretical Framework Integration

The mediation of productive failure effectiveness by metacognitive awareness is worth particular attention (Güneş, 2022). Studies that have shown that interactive learning with errors can facilitate the development of metacognition indicate that the effects of productive failure is not only limited to domain-area specific mathematical knowledge but also extend to broad-based learning skills such as error identification and strategy monitoring in addition to adaptive thinking (Metcalf et al., 2024). Such metacognitive acquisitions can be especially useful regarding long-term mathematical growth because students with advanced self-regulation techniques have a higher ability to learn on their own and handle the problem-solving process in new circumstances. Nevertheless, understanding how the engagement in errors can be converted into improved metacognitive awareness is an ultimate part of further research, as it is necessary to understand what sort of reflection and analysis turns out to be the most fruitful among various learners and mathematical areas (Schoenfeld, 2016).

Implementation Challenges and Professional Development Needs

The high imprecision in the teacher implementation fidelity that has been reported in the research entails a significant obstacle to embedding the research on productive failure into the practise in the classrooms (Vedder-Weiss et al., 2018).

The implementation process requires pedagogical proficiency that involves more than one dimension such as task design, the promotion of productive struggle, classroom discourse, and strategic consolidation by tapping into the student thinking (Kang et al., 2025). The teachers should learn to have advanced ideas concerning when to permit struggle to persist and when to assist, how to sustain positive emotional environments and yet sustain intellectual challenge and how to expand upon the student generated solutions when consolidating in a manner that leads to the development of conceptual knowledge. These competencies are challenging with the use of the conventional model of professional development with a focus on the passing of the theoretical knowledge, but continue to be the one that requires long-term experience-building through the availability of the beneficial failure experience during which teachers serve to be learners in the interactive learning practises and undergo coaching that supports them in the analytical reflection on their pedagogical choices (Kramarski, 2017). The difference between productive and unproductive struggle appears to be of specific significance in the achievement of implementation success. Struggle does not always enhance learning, and overcompensation, spinning of the process without a conceptual interaction and encouragement of false beliefs are some examples of unproductive challenge that impedes, but does not benefit learning (Paurowski et al., 2024). To successfully implement the idea of productive failure, the skillful consideration of task design, with the purpose of ensuring that the challenges provoke the corresponding background knowledge, without making them harder to solve using it, and also, being available enough to facilitate authentic involvement, social supports, which may be provided by collaborative settings that maintain motivation and offer scaffolding, and teacher responsiveness, which identifies and responds to unproductive patterns, are essential. Teacher preparation programmes that include practical learning of the productive struggle strategies are promising in the development of these competencies though there is little longitudinal research that considers the sustainability of the quality of implementation in the long run.

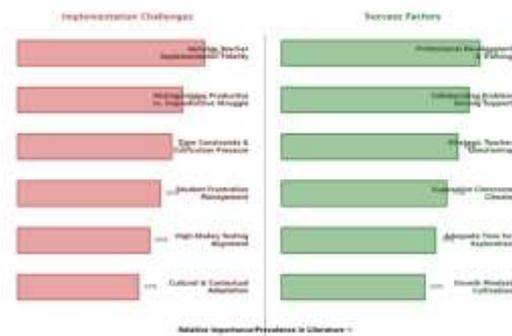


Figure 5: Implementation Challenges and Success Factors
Individual Differences and Equity Considerations

Results of individual differences in productive failure effectiveness contradict previous beliefs that exploration-based instruction was only able to help high-achieving students or children with high prior knowledge of mathematics. Experimental studies that have found demonstrations that inventive production capacity as well as learning gains can be found at different levels of achievement implies greater inclusivity than critics had initially presumed (Kapur et al., 2023). Nevertheless, there are still significant questions about that the effectiveness of productive failure is dependent on the characteristics of learners (such as previous mathematical experiences, self-efficacy, cultural backgrounds and learning preferences). The possibilities of fruitful forms of failure working toward tightening or alleviating achievement gap warranted close empirical focus and so does the exploration of how implementation can be adapted so as to serve diverse learners at the same time of retaining the fundamental tenets of exploration and struggle.

Motivational and affective aspects are also key factors to consider when implementing productive failure education based on logic of equity and sustainability. Empirical studies of transformational changes in the attitudes of students towards errors and establishment of developmental mindset orientations propose potentially significant affective benefits that can be considered complementary to cognitive ones (Luzano, 2025). These motivational results can be especially useful when working with students who had negative experiences in math before or who believe in the fixed

mindset factor that can hinder learning and engagement. Nevertheless, such risks as high frustration, anxiety, and learned helplessness should be addressed proactively, where the psychological safety, the normalisation of struggle, and the glorification of effort and improvement progress in addition to success should be implemented. Striking a balance between providing challenge enough to facilitate cognitive activation, and not overwhelming frustration that destroys motivation is a fragile process demand of teacher attentiveness and sleepiness to the needs of an individual student.

Technology Integration: Opportunities and Challenges

Further lack of integration of technology in basic productive failures research would be a gap because of the promise that digital tools have to contribute to exploration and visualisation as well as collaboration. Dynamic geometry systems allow testing conjectures very quickly and visualising mathematical relationships that would otherwise be a laborious and laborious task in the context of paper-and-pencil tests only (Santos-Trigo, 2019). Programming environments enhance solution strategies by allowing an iterative approach to refinement by providing a feedback on solution strategies and documenting problem solving mechanisms. Computer algebra systems minimise the computational loads in order that cognitive resources and strategic thought on the part of a learner are diverted towards conceptual relations and relations. Nevertheless, to obtain these possible advantages, careful consideration and incorporation is needed that will not destroy the mental activity necessary to achieve effective productive failure.

There is the threat that technology is going to eliminate instead of make productive struggle productive. Digital aids that automate excess of the problem-solving procedure or offer scaffolding that short circuits creative production might decrease the activation and differentiation of knowledge that are the primary mechanisms of productive failure. The ideal technology involvement may comprise the ones that increase the range of available problem solving methods and displays but do not decrease cognitive demand and learner agency. Studies on the nature of interaction between various technological

affordances and the principles of productive failure are scarce yet they are an avenue that deserves further research particularly given the growing capabilities in artificial intelligence and adaptive learning engines to mediate mathematical learning processes.

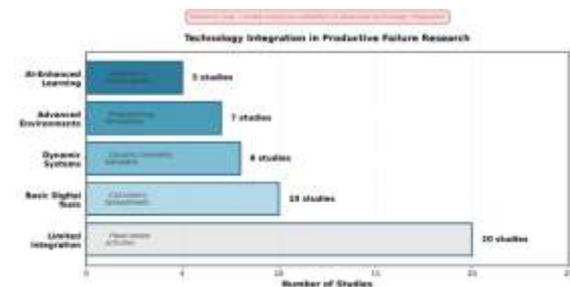


Figure 6: Technology Integration Continuum
Contextual Factors and Cultural Considerations

The localization of research of productive failures to same settings, especially Singapore and Korea, brings up the issue of cultural transferability and situational requirements (Lee & Hung, 2016). Educational cultures with more emphasis on the elements of collaborative learning, tolerance of struggle and student-centred pedagogy might offer more welcoming conditions of productive failure execution compared to the contexts of teacher authority, error free performance and competitive accomplishment. Cultural values pertaining to the concept of mathematical ability in general, the role of hard work versus innate cognition, and the correct teacher student relationship can impact both teacher disposition to make use of productive failure strategies and the student willingness to adopt instruction through exploration. Research on the variation in the effect of productive failure in different cultural and institutional contexts, conducted systematically, would enhance the knowledge of the parameters of the boundary conditions and lead to the change strategies of various learning environments (Bouw et al., 2019).

The productive failure implementation feasibility and sustainability also mediate institutions factors such as assessment systems, curriculum requirements and administrative support. Large-stakes testing that focuses on procedural fluency and fast problem-solving can put a strain on the efficiency of instruction in conflict with time

commitment of discovery and integration (Sorooshian, 2017). The demanding curricula with little time left to engage in elaborate problem solving processes are potential limiting factors to the executions of productive failure by the teacher even with pedagogical dedication (Vedder-Weiss et al., 2018). Policies and accountability frameworks acknowledging and giving credit to deep conceptual knowledge as well as procedural expertise would establish better conditions favouring productive adoption of failure and long term implementation (Scarborough et al., 2015).

Conclusion

This overarching synthesis of the research conducted on productive failure and desirable difficulties proves that these pedagogical strategies are strong tools to increase the long-term retention, transfer, and problem-solving skills in mathematics education. The empirical compilations of the literature are always recorded of stronger conceptual knowledge and versatile use of the math knowledge accessible after the productive failure teaching than the conventional direct teaching methods, but does not affect procedural fluency. Combining constructivist learning theories, cognitive load theory and motivation models theoretically, gives strong explanatory theories, which can explain the underlying cognitive, metacognitive, and affective processes to these benefits.

Nevertheless, it requires advanced pedagogical skills in the classroom practise to achieve the benefits of productive failure that include shaping work, enabling productive conflict, and consolidating the wisdom of the student thinking. The inconsistency of implementation fidelity underscores the need to have an ongoing professional growth that assists teacher in developing such competencies. Research gaps should surround technology integration issue, cultural transferability, and individual differences, and quality sustainability of implementation in future studies. Nonetheless, fruitful failure and desirable difficulties present good opportunities to change the way mathematics is taught and learn to methods that can lead to the development of in-depth knowledge, innovative problem solving and versatile skills that can be acquired to meet the demands of more and more sophisticated fields in mathematics and science. Teachers

and policy makers are urged to explore these evidence based strategies in developing teaching and learning systems of mathematics in the form of instruction models and professional development models.

References

- Abramovich, S., & Cho, E. K. (2012). Technology-enabled mathematical problem posing as modeling.
- Al-Jarrah, I., & Melhem, A. (2024). The effect of using the productive struggle strategy on improving conceptual understanding in mathematics among fifth grade students. <https://doi.org/10.59992/ijesa.2024.v3n3p4>
- Ajani, O. A. (2023). Teachers' professional development in the context of experiential learning theory for improved instructional delivery. *African Journal of Development Studies*, 13(4), 131.
- Bouw, E., Zitter, I., & De Bruijn, E. (2019). Characteristics of learning environments at the boundary between school and work—A literature review. *Educational Research Review*, 26, 1-15.
- Casing, P. I., & Roble, D. B. (2021). Students' mathematical creative thinking ability with posing-exploring-doing-evaluating (pede) productive failure model in new normal. *American Journal of Educational Research*, 9(7), 443-448. <https://doi.org/10.12691/EDUCATION-9-7-8>
- Chen, O., Chen, O., & Kalyuga, S. (2020). Exploring factors influencing the effectiveness of explicit instruction first and problem-solving first approaches. *European Journal of Psychology of Education*, 35(3), 607-624. <https://doi.org/10.1007/S10212-019-00445-5>
- Coelho, A., & Cabrita, I. (2017). Creativity enhanced by technological mediation in exploratory mathematical contexts. https://doi.org/10.1007/978-3-319-61322-2_3
- Freiman, V., & Tassell, J. L. (2018). Leveraging mathematics creativity by using technology: Questions, issues, solutions, and innovative paths. https://doi.org/10.1007/978-3-319-72381-5_1
- Felton, M., Garcia-Mila, M., Villarroel, C., & Gilabert, S. (2015). Arguing collaboratively: Argumentative discourse types and their potential for knowledge building. *British Journal of Educational Psychology*, 85(3), 372-386.

- Güneş, Ö. (2022). Failure attributions and metacognitive awareness of EFL learners. *Language awareness, 31*(1), 53-72.
- Huang, X., & Zhou, J. (2024). Action research on the teaching model of deep learning in the smart classroom based on the perspective of productive failure. *Communications in Computer and Information Science, 174*-194. https://doi.org/10.1007/978-981-99-9492-2_16
- Isaacs, R. (2025). Exploratory learning before instruction: Comparing individual exploration, collaborative exploration, and preparation for future collaboration.
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in mathematics, 96*(1), 1-16.
- Jeyabal, K., Vasuki, M., & Kumar, A. D. (2015). A Critical Examination of Problem-Based Learning in Mathematics Education. *Journal of Engineering Scientific Research and Applications, 1*(2), 154-164.
- Kalyuga, S., & Plass, J. L. (2025). Empirical evidence for the goal-driven approach. <https://doi.org/10.1093/9780190078539.003.0006>
- Kang, H. J., Mahovsky, K. A., & Harding-Middleton, J. (2025). Learning to teach productive struggle: Elementary preservice teachers' insights from modeling in mathematics. *International Journal of Educational Studies, 8*(6), 26-36. <https://doi.org/10.53935/2641-533x.v8i6.531>
- Kapur, M. (2011). A further study of productive failure in mathematical problem solving: Unpacking the design components. *Instructional Science, 39*(4), 561-579. <https://doi.org/10.1007/S11251-010-9144-3>
- Kapur, M. (2012). Productive failure in learning the concept of variance. *Instructional Science, 40*(4), 651-672. <https://doi.org/10.1007/S11251-012-9209-6>
- Kapur, M. (2014). Comparing learning from productive failure and vicarious failure. *The Journal of the Learning Sciences, 23*(4), 651-677. <https://doi.org/10.1080/10508406.2013.819000>
- Kapur, M. (2015). Learning from productive failure. <https://doi.org/10.1080/23735082.2015.1002195>
- Kapur, M. (2016). Examining productive failure, productive success, unproductive failure, and unproductive success in learning. *Educational Psychologist, 51*(2), 289-299.
- Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *The Journal of the Learning Sciences, 21*(1), 45-83. <https://doi.org/10.1080/10508406.2011.591717>
- Kapur, M., Saba, J., & Roll, I. (2023). Prior math achievement and inventive production predict learning from productive failure. *npj Science of Learning, 8*(1). <https://doi.org/10.1038/s41539-023-00165-y>
- Kramarski, B. (2017). Teachers as agents in promoting students' SRL and performance: Applications for teachers' dual-role training program. In *Handbook of self-regulation of learning and performance* (pp. 223-239). Routledge.
- Lee, S.-S., & Hung, D. (2016). A socio-cultural perspective to teacher adaptivity: The spreading of curricular innovations in Singapore schools. *Learning: Research and Practice, 2*(1), 64-84
- Lee, D., & Lee, Y. (2024). Productive failure-based programming course to develop computational thinking and creative problem-solving skills in a Korean elementary school. *Informatics in Education, 23*(1). <https://doi.org/10.15388/infedu.2024.14>
- Luzano, J. F. (2025). Mistakes as missed takes: Unlocking the transformational power of productive-failure strategy in mathematics education. *Social Science Research Network, 2025*. <https://doi.org/10.2139/ssrn.5479246>
- Metcalf, J., Xu, J., Vuorre, M., Siegler, R., Wiliam, D., & Bjork, R. A. (2024). Learning from errors versus explicit instruction in preparation for a test that counts. *British Journal of Educational Psychology, 95*(1). <https://doi.org/10.1111/bjep.12651>
- Ngu, B. H., & Phan, H. P. (2024). Instructional approach and acquisition of mathematical proficiency: Theoretical insights from learning by comparison and cognitive load theory. *Asian Journal for Mathematics Education, 2024*. <https://doi.org/10.1177/27527263241266765>
- Olivares, R. G., Olmo, M., Muñoz, J. D., González, J. D., Calero, J. A., & Arnau, D. (2024). Analysing the impact of erroneous examples on third-grade students' problem-solving proficiency. *International Journal of Innovation in Science and Mathematics Education Research, 2024*. <https://doi.org/10.1080/10508406.2024.2373508>

- Education, 32(3).
<https://doi.org/10.30722/ijisme.32.03.005>
- Paurowski, M., Glassmeyer, D., Kim, J., & Id-Deen, L. (2024). Struggling as part of success: International baccalaureate students' productive struggle is strongly correlated to mathematical achievement. *International Journal of Mathematical Education in Science and Technology*, 1-18.
<https://doi.org/10.1080/0020739x.2023.2296583>
- Santos-Trigo, M. (2019). Mathematical problem solving and the use of digital technologies. https://doi.org/10.1007/978-3-030-10472-6_4
- Santos-Trigo, M., Aguilar-Magallan, D. A., & Reyes-Martinez, I. (2019). A mathematical problem-solving approach based on digital technology affordances to represent, explore, and solve problems via geometric reasoning. https://doi.org/10.1007/978-3-030-29215-7_8
- Santos-Trigo, M., Reyes-Martinez, I., & Aguilar-Magallan, D. A. (2016). Digital technologies and a modeling approach to learn mathematics and develop problem solving competencies. https://doi.org/10.1007/978-3-319-42147-6_17
- Scarborough, H., Robertson, M., & Swan, J. (2015). Diffusion in the face of failure: The evolution of management innovation. *British Journal of Management*, 26(3), 365-387.
- Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of education*, 196(2), 1-38.
- Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding mathematical modelling with a solution plan. *ZDM*, 47(7), 1241-1254.
- Sinha, T., Kapur, M., West, R., Catasta, M., Hauswirth, M., & Trninic, D. (2021). Differential benefits of explicit failure-driven and success-driven scaffolding in problem-solving prior to instruction. *Journal of Educational Psychology*, 113(3), 530-555.
<https://doi.org/10.1037/EDU0000483>
- Sorooshian, S. (2017). Author's index. *KOME: An International Journal of Pure Communication Inquiry*, 5(2), 71-72.
- Stronge, J. H., & Xu, X. (2021). *Qualities of effective principals*. ASCD.
- Vedder-Weiss, D., Ehrenfeld, N., Ram-Menashe, M., & Pollak, I. (2018). Productive framing of pedagogical failure: How teacher framings can facilitate or impede learning from problems of practice. *Thinking Skills and Creativity*, 30, 31-41.
- Westbrook, A. F. (2014). A case study of how ninth grade mathematics students construct knowledge during a productive failure model.
- Wu, B. (2024). Decomposing the cognition-influential aspects of instructional designs a systematic review of induced causal relationships by the constructional components of the direct instruction and productive failure designs. *Advances in Social Science and Culture*, 6(5), p100-p100.
<https://doi.org/10.22158/assc.v6n5p100>